EM-I

Final Exam for Electromagnetic I

2 hours (29/02/2024)

Answer all questions

(Note: Open Book Exams, Don't use answers notes, don't use your friend tools, and don't use the computer)

Q1

Express as a vector function the gradient (maximum directional derivative) of the following scalar fields

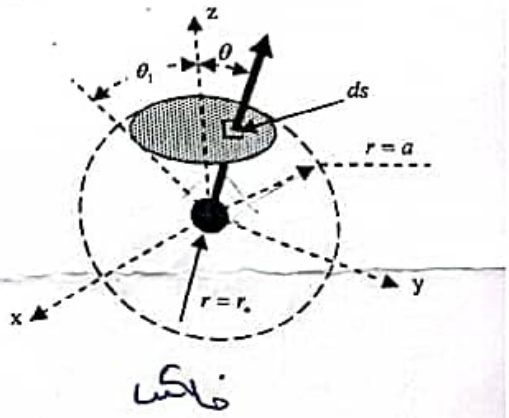
$E(\rho,\varphi,z) = 15\rho^2\cos\varphi + 200\rho\sin\varphi + 100z\cos\varphi$

$G(\rho,\varphi,z) = 75\rho^2 z\cos\varphi + 50\rho\sin\varphi$

Find the magnitude of ∇E and ∇G at $p_1(2,45^\circ,1)$ and determine the expression for the unit vector $a_{\nabla E}$ and $a_{\nabla G}$. Find the angle between ∇E and ∇G at $p_1(2,90^\circ,1)$.

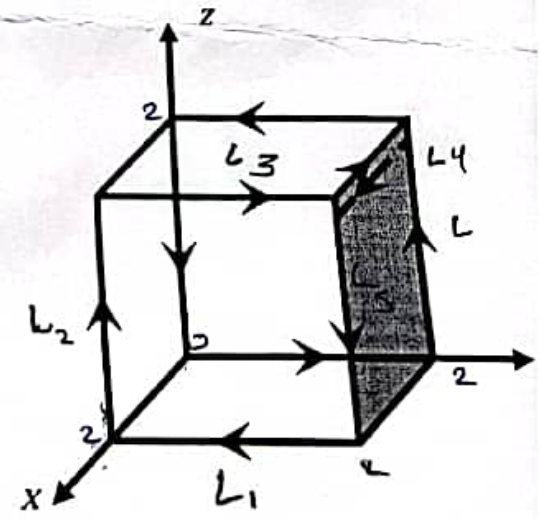
Q2

Given is the E-field solution $E_r = \frac{\rho_v r_o^3}{3\varepsilon_o r^2}$ for the volume charge $Q_v(r = r_o)$ located at the origin. Imagine the spherical surface of radius $r = a$ to surround the charge. *(a)* Evaluate the flux ψ of the vector $\varepsilon_o E_r$ passing through the surface of the spherical cap bounded by $\theta = \theta_1$ as shown. *(b)* Use the result of *(a)* to find the angle $\theta = \theta_1$ that gives maximum value of $D_{max} = \varepsilon_o E_r$.



Note: $\psi = \int_s F \cdot ds$

Q3 Given the vector field $G(x,y,z) = a_x 10x^3yz^2$, *(a)* Determine whether or not vector G has flux source? *(b)* for the cubic as shown in figure 2, illustrate the validity of the divergence theorem by evaluating the volume and the surface integral inside and on the cubic bounded by coordinate surfaces x=0,x=2, y=0,y=2, z=0, z=2 for the given G. (c) use the path as shown in the figure to evaluate the equation below.



$$\int_S (\nabla \times \mathbf{G}) \cdot ds = \oint_\ell \mathbf{G} \cdot d\ell$$

Good luck